TEACHING UNIT FOR THE ARKANSAS SCHOOL FOR MATH AND SCIENCES

APPLICATIONS OF THE LOGARITHM

Earthquakes, pH and Decibels

Dr. Charles Mullins, Math Dept
Email: mullinsc@asms1x.dsc.k12.ar.us

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ASMS; Email: mullinsc@asms1x.dsc.k12.ar.us

OBJECTIVES:

1) Students will see the use of logarithms in the formula for the Richter scale and be able to use the formula to obtain numerical results relating magnitude and movement of the earth's surface.
2) Students will learn about and use the formula that defines the pH of a solution in terms of the logarithm of the concentration of Hydrogen atom in the solution. They will use this formula to answer the question: Are solutions with high pH more or less acidic?
3) Students will learn how to measure audible sound in decibels using a logarithmic scale and the relation between decibels and the intensity of sound.

KEY POINTS:

Applications of logarithms to the Richter scale for earthquakes, pH for solutions, and decibels for sound.

1) The relationship between magnitude R of an earthquake using the Richter scale and the amplitude a of the seismic wave measured in microns μ = one millionth of a meter and corresponds to the vertical movement of the earth's crust: $R = \log(a/T) + b$ where T is time in seconds and b is a constant determined "experimentally".
2) The pH of a solution is given by: $pH = -\log(H+)$ where H+ is the concentration in moles of Hydrogen ions in the solution. Acids have a pH < 7 = to the pH of distilled water.
3) To measure a sound in decibels D, we compare the sound's intensity I, to the intensity of a standard benchmark $I_o = 10^{-16}$ watts/cm^2, which is roughly the lowest intensity audible to humans: $D = 10 \times \log(I/I_o)$

PRESENTATION:
EARTHQUAKES (1 DAY)

a) Generate class discussion with: Has anyone been in a big earthquake? Discuss various earthquakes and that they are measured using the Richter scale R.

b) Use the attached sheets to highlight the Richter scale. An Internet search on earthquakes will produce many other sites.

c) Formula for the Richter scale: The magnitude of an earthquake is the number R where

$$R = \log(a/T) + b$$

a = amplitude (in microns μ = one millionth of a meter) of the seismic wave which indicates vertical ground movement, T is the period (in seconds) of the seismic wave and B is a constant which is determined "experimentally". In this paper we will take T = 2.5 seconds and B = 4.240.

d) Using this formula determine the amount of vertical movement of the earth?
Example 1. The New Madrid earthquake in 1811-12 of magnitude 8.4-8.7 on the Richter scale (use magnitude 8.7).
Note: The New Madrid fault runs through Illinois and then along the Mississippi River in Missouri and Arkansas and includes Crowley's Ridge. One reference indicates that the magnitude of this earthquake may have been only about 7.4.
Solution: $\log(a/2.5) + 4.240 = 8.7 \Rightarrow \log(a/2.5) = 4.460 \Rightarrow a/2.5 = 10^{4.460} \Rightarrow$
$a = 2.5\text{x}10^{4.460} \approx 72{,}100$ microns $= 72{,}100\text{x}10^{-6}$ meters $= 0.0721$ meters $= 7.21$ cms $\approx$ $7.21/2.54$ inches ≈ 2.839 inches.

Example 2. The San Francisco earthquake of 1906 of magnitude 8.3 on the Richter scale.
Solution: Replacing 8.7 by 8.3 above gives $a = 2.5\text{x}10^{4.060} \approx 28{,}700$ microns $= 0.0287$ meters $= 2.87$cms ≈ 1.130 inches.

Example 3. Any of the recent earthquakes in the news, e.g. on November 12 Turkey experienced an earthquake of magnitude 7.2 on the Richter scale.
Solution: If R is the magnitude of the earthquake, $\log(a/2.5) + 4.240 = R \Rightarrow \log(a/2.5) = R - 4.240 \Rightarrow a/2.5 = 10^{(R-4.240)} \Rightarrow a = 2.5\text{x}10^{(R-4.240)}$. Hence, for $R = 7.2$ we obtain $a = 2.5\text{x}10^{(7.2-4.240)} \approx 2{,}280$ microns $= 0.00228$ meters $= 0.228$ cm ≈ 0.0898 inches.

Assignment:
1. Do an Internet search on earthquakes to learn more about earthquakes.
2. Determine the amount of movement of the earth for some of the earthquakes listed in the Internet search or in the attached sheets.
3. Compare the magnitude of various earthquakes.
General solution: If R_1 and R_2 are the magnitudes of two earthquakes and a_1 and a_2 are the amounts the earth moved, respectively, then choosing R_2 to be the larger value,
$R_2 - R_1 = (\log(a_2/2.5) + 4.420) - (\log(a_1/2.5) + 4.420) = \log(a_2/2.5) - \log(a_1/2.5) = \log((a_2/2.5)/(a_1/2.5)) = \log(a_2/a_1) \Rightarrow a_2/a_1 = 10^{(R - R)}$. For example, the movement for an earthquake of magnitude $R_2 = 9$ is 1000 times as much as than for one of magnitude $R_1 = 6$.

ACIDITY AND pH. (1/2 Day)

Question: What do we mean by the pH of a solution? What does it measure?
Definition: A Measure of Chemical Acidity. Formula: pH = -log (H+) where
H+ = concentration of Hydrogen ions in the liquid measured in "moles per liter".

a) What is the pH for seawater which has concentration of Hydrogen ions H+ = 1.1x10 (= 1.1E-8 on the calculator)?
Solution: For our purposes we use the value for H+ of $1\ \text{x}10^{-8}$ since to one decimal place the answers will be the same. $\text{pH} = -\log(1\text{x}10^{-8}) = -\log(10^{-8}) = -(-8) = 8$.
Here it is important to recall that $\log(10^n) = n$ for any n.

b) What is the ion concentration of a vinegar solution with a pH of 3? How does it compare to seawater?
Solution: $-\log (H+) = 3 \Rightarrow H+ = 10^{-3} = 1/10^{3} = 1/1000 = .001$ moles per liter (or we could just write $10^{-3} = 0.001$).
Recall that the concentration of Hydrogen ions in vinegar was approximately 10^{-8} moles per liter; so, the concentration of Hydrogen ions in vinegar is nearly $10^{-3}/10^{-8} = 10^{(-3-(-8))} = 10^{5} = 100{,}000$ times that of seawater.

Assignment:
1. Ammonia has a pH of 10. What is its Hydrogen ion concentration?
Solution: $-\log (H+) = 10 \Rightarrow H+ = 10^{-10}$ moles per liter = 0.0000000001 moles per liter.

2. What is the pH of distilled water which has a concentration H+ = 0.0000001 moles per liter?
Solution: $0.0000001 = 10^{-7}$ so the pH $= -\log (10^{-7}) = -(-7) = 7$.

3. Solutions are divided into two classes: acids and bases depending on whether their pH is above or below that of distilled water. Examining the results above is the pH of acids above or below 7?
Solution. Since vinegar is an acid, it follows that acids have a pH < 7.

DECIBELS (1/2 Day)

The decibel scale was designed to reflect human perception of how sound changes and studies indicate that it is related to the logarithm of the change in intensity. Noise levels are measured in units called decibels in honor of the inventor of the telephone, Alexander Graham Bell. To measure a sound in decibels, we compare the sound's intensity I to the intensity of a standard benchmark sound I_o. The intensity of this benchmark sound I_o is defined to be 10^{-16} watts/cm^2 and is roughly the lowest intensity audible to humans. The comparison between a given sound intensity I and the benchmark sound intensity I_o is given by the following expression:

$$\text{noise level in decibels} = 10 \times \log(I/I_o)$$

The expression I/I_o gives the relative intensity of sound compared to the benchmark I_o.

a) The level of typical conversation is 50 decibels. What is the intensity of this sound?
Solution. According to our formula above, if I is the intensity of conversation, then

$$10 \times \log (I/I_o) = 50 \Rightarrow \log (I/I_o) = 5 \Rightarrow 10^{5} = I/I_o \Rightarrow$$

$$I = 10^{5}I_o = 10^{5}(10^{-16}) = 10^{(5-16)} = 10^{-11} \text{ watts/cm}^2.$$

b) What is the decibel level of a typical rock band playing with an intensity of 10^{-5} watts/cm^2? How much more intense is the sound of the band than an average conversation?

Solution: Let D represent the decibel level. Then D = 10 x log ($10^{-5}/10^{-16}$) =

10 x log ($10^{-5-(-16)}$) = 10 x log (10^{11}) = 10 x 11 = 110 decibels.

We know ordinary conversation has an intensity of 10^{-11} watts/cm^2 so the ratio of the intensity of the rock band to ordinary conversation is $10^{-5}/10^{-11} = 10^{-5-(-11)} = 10^6$ = 1,000,000. Thus the rock band produces a sound about 1 million times as intense as ordinary conversation.

Assignment:

1. What is the decibel level of a sound whose intensity is $1.5x10^{-12}$ watts/cm?

Solution. Using our formula above, we obtain

10 x log ($1.5x10^{-12}/10^{-16}$) = 10 x log ($1.5x10^4$) ≈ 10 x 4.2 = 42 decibels.

2. If the intensity level increases by a factor of 100, what is the increase in the decibel level? What if the intensity is increased by a factor of 10 million?

Solution. If a sound has a decibel level D_1 or D2 corresponding to an intensity I_1 or I_2, respectively, then D_1 = 10 x log (I_1/I_o) and D_2 = 10 x log (I_2/I_o). If D_2 denotes the larger of the two decibels levels, then $D_2 – D_1$ = 10 x log (I_2/I_o) – 10 x log (I_1/I_o) =

10 x log ((I_2/I_o)/(I_1/I_o)) = 10 x log ((I_2/I_o)x (I_o/I_1)) = 10 x log (I_2/I_1) . Thus, if the noise level I_2 is 100 times as intense as that of I_1, then $I_2 = 100 I_1 \Rightarrow I_2/I_1 = 100 \Rightarrow$

$D_2 – D_1$ = 10 x log (100) = 10 x log (10^2) = 10 x 2 = 20. Therefore the decibel level increases by 20. If the noise level I_2 is 10,000,000 times as intense as I_1, then

$I_2/I_1 = 10,000,000 = 10^7 \Rightarrow$ $D_2 - D_1$ = 10 x log (10^7) = 10 x 7 = 70. Thus the decibel level increases by 70.

3. Compare the intensity of some of the following sounds with the indicated decibel levels.

(i) Whisper, 20 decibels (ii) Average radio, 70 decibels (iii) Loud truck, 90 decibels

(iv) Jackhammer or jet airplane, 120 decibels

(v) Threshold of pain where ears hurt, 130 decibels

The following can be found at: http://cbcnews.cbc.ca/news/indepth/earthquake/

Measuring an earthquake

The first practical scale for measuring earthquakes was developed by geologist Charles Richter at the California Institute of Technology in the 1930s, and the scale that most scientists use today still bears his name. (Actually, seismologists use several different, but related, scales. But the Richter scale, denoted by a number called the "magnitude," is the most common. This quantity, which can be read off a Seismograph, reflects the amount by which the earth's crust shifts.) The Richter scale has no lower limit and no maximum. It's a "logarithmic" scale, which means that each one-point increase on the scale represents a ten-fold increase in the magnitude of the quake.

However, the energy released by an earthquake increases at an even steeper rate, going up by a factor of 32 for each one-point increase in magnitude.

A quake with magnitude between 2 and 3 is the lowest normally felt by people. A magnitude 5 quake is considered moderate; less intense quakes rarely cause any damage. Worldwide, there are about 1,500 earthquakes of magnitude 5 or higher every year. An earthquake of magnitude 6 or higher is considered major.

The largest earthquakes in history have been of about magnitude 9. Major earthquakes release far more energy than any man-made explosion. The 1906 San Francisco earthquake, with a magnitude of 8.3, was approximately one million times as powerful as the atomic bomb dropped on Hiroshima.

Notable earthquakes of the 20th century

Date	Location	Magnitude	Deaths
April 18, 1906	San Francisco	8.3	503
Aug 16, 1906	Chile	8.6	20,000
Dec. 16, 1920	China	8.6	100,000
Sept. 1, 1920	Japan	8.3	100,000
May 22, 1927	China	8.3	200,000
March 2, 1933	Japan	8.9	2,990
Jan. 15, 1934	India	8.4	10,700
Jan. 24, 1939	Chile	8.3	28,000
May 21-30, 1960	Chile	9.5	5,000
July 28, 1976	China	7.8 to 8.2	242,000
Sept. 19, 1985	Mexico	8.1	9,500
June 21, 1990	Iran	7.3 to 7.7	50,000