A Teaching Unit in Applied Science

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Teaching Unit – Exercises in Applied Science Problem Solving

Goals and Objectives:

Students will significantly increase their "real world" problem solving capacity through a relatively (to most school exercises) lengthy effort in mastering problems with characteristics that approach real world problem characteristics. Refer to page 2 attached for some of these characteristics.

Prerequisites and appropriate level of difficulty/student level:

Though designed for programming courses at the senior high level, this unit could be mastered by bright students with access to programming software (C++, Basic, Pascal, etc.) and some modicum of programming experience.

Approach:

This unit should be a coached and guided experience with some of the best left over techniques of problem or project based learning. (Note that "techniques" implies teaching, but the word learning is used!)

Time/sessions required:

Modifiable by student abilities and progress, approximately one week, three sessions, 3-5 hours.

Sequence of Activities:

<u>First session/block</u>- the teacher will distribute the handout "The Great Dunce Hat Dump" (page 3 attached). Students are to read the handout and outline and/or discuss in teams how the problem might be solved. As soon as simulation by computer is recognized as the viable alternative, the teacher will discuss interactively the geometry and logic of page 4.

<u>Session/block 2</u>-students will work individually or in teams of two maximum to encode the problem as demonstrated on page 5. Note that both tanks need not be simulated in the same program as on page 5.

<u>Session/block 3</u>-students will run their simulations, post the results, and connect these results to the final best solution of the problem as demonstrated on page 6.

<u>*Assessment*</u>:

The problem on page 7 will perform evaluation of the students' ability to simulate a given problem and make logical conclusions based on the results. The first two questions test the ability to simulate and obtain output. The second two further test the students' reasoning ability, which should have been enhanced by the viscosity aspect of the tank problem. Good students will not predict that a 2.5 minute delay is workable since the trip required 7.5 minutes out of 10 allowed.

Note: These problems have proven stimulating and interesting to students.

The "DunceHat" Problem Characteristics

1. Range of possible correct answers, some outside initial expectations, some better than others.

2. Extraneous information is given- some obvious, some not.

3. Some data appears to be relevant before closer scrutiny.

4. Necessary information is not given in order of utilization.

5. Problem is multilevel- reading comprehension, geometry, algebra, programming, common sense.

6. Scratch padding and mapping to the side are generally required.

7. Multiple readings and "rescanning" are required by most.

8. Solution requires more time than does the average test problem.

9. All information is not given in immediately usable form.

10. Requires interpretation and organization.

12. Requires persistence and precision.

13. Programming code required is simple yet sophisticated and powerful.

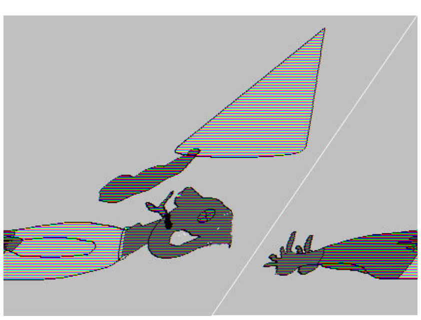
The Great Dunce Hat Dump

Two ancient, alien, chemical, conical tanks were found, one near Mount Gimbo just outside of Putz, Panama (sea level, population 1492), and another on the east side of Catapultic Gulch near Cuzman, Peru (elevation 13500 ft.). Refer to the Gimbo tank as tank 1 and the Cuzman tank as tank 2. Both tanks are situated on flat ground with the large round ends down, as opposed to balanced on their pointy little tops. Tank 1 is 10 feet high and 8 feet in diameter at the base. Tank 2 is 4 feet high and 5 feet in diameter at the base. Tank 2 contains a catalytic synergizer for the mix in tank 1. If both tanks were emptied, the effluvients would mix in the atmosphere and cause exponential collapse of the earth=s oxygen mantle. If either tank were to retain any of its contents, the result would be harmless.

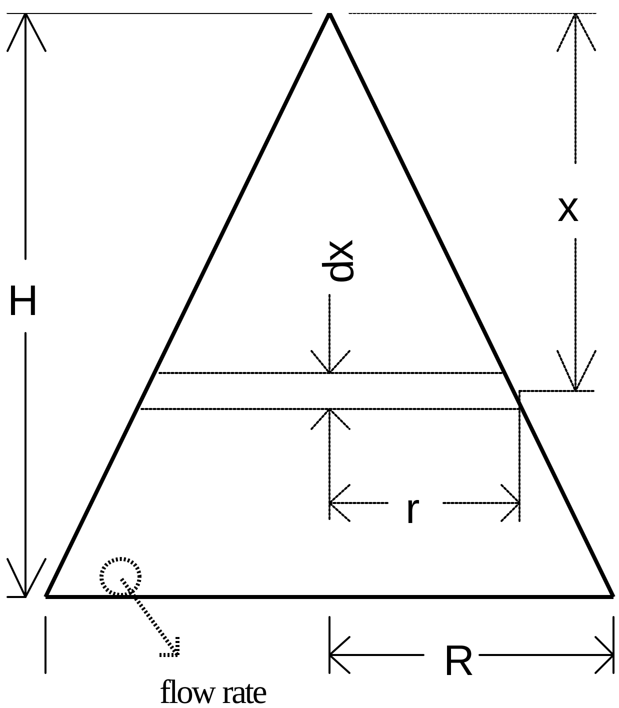
At 6 o=clock this evening mutants with alien genes opened holes in the bottom edge of both tanks before United Nations troops dispatched them humanely. Detailed information, oddly readable, was found on their persons (well, bodies then). It is now 7:16 in the evening and the following is known:

The only entities on earth with the technology and general all around wherewithal to seal the tank leaks are the dynamic duo of Raquel Whizman and her nerd/athlete sidekick Yubbo Dorfman, who both reside right here near Lake Hamilton. The trip to the Panama site requires 4 hours and 14 minutes while the trip to the Peru site will take 7 hours and five minutes. With the following information about the leaks, should the pair go to Peru, to Panama, stay home and watch ESPN, or apply for the two remaining volunteer openings in the biodome?

The density of the fluid in tank 1 is 212 pounds per cubic foot and that in tank 2 is 50 pounds per cubic foot. The rate of flow from tank 1 is given by 16+10*p-7.5*v cubic feet per hour, where p is in pounds per square inch. For example, the pressure p for tank 1 when there are two feet of fluid left is (2 x 212)/144 pounds per square inch. V is viscosity in kopters. Note you don=t need to know what viscosity means so you won=t worry about kopters, either. The negative viscosity factor impedes outflow. Viscosity in both tanks is 0.21*t kopters, t in hours after tank breach. Tank 2 orifice characteristics are such that its flow rate formula is 3.2+5*p-1.7v. Tank 1 was filled to 7.5 feet and tank 2 to 3 feet.



Conical Tank Emptying Simulation



H, R, initial x, and the flow rate expression are given. The flow rate may be in terms of time or any other variable shown. The flow rate is a measure of volume per time.

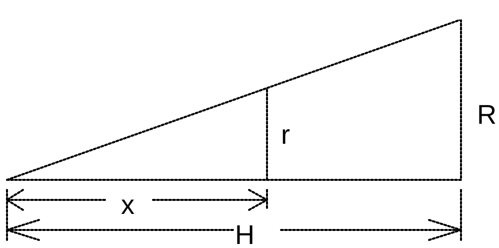
dx is the change in x which occurs in time dt. dt is the small increment in time which steps the simulation. It is small enough that dx and other changes in variable are very small. Indeed r can be considered the same before and after the dx increment. Herein is the power in incremental simulation.

The key recognition (leap maybe) is that the volume of the cylinder of height dx is equal to the flow rate times the time increment, i.e., the volume lost through the hole (leak) in time dt is equal to the volume of the cylinder of height dx.

The programming segment required for the simulation is:

```
x=2.5;
        while (x<10)
        {
                t+=dt;
                p=212*(10-x)/144;
                v=0.21*t;
                dvol=(10+10*p-7.5*v)*dt;
                if (dvol<=0)
                break;
                dx=dvol/(3.1417*r*r);
                x+=dx;
                r=(0.4)*x;
        }
        cout<<t<<" "<<"Panama"<<" "<<dvol<<" "<<x;
```

A key relationship above is the relationship of r to x via similar triangles and the resulting proportions. (see below) The "dvol" variable is simply the volume of the cylinder of height dx as discussed above. Its first occurrence in the code is simply its flow rate equivalent. Then dvol is divided by the area of the wafer cylinder of thickness dx to yield the small change in x (dx).



The full program which executes both tank simulations

```
#include <iostream.h>
main()
{
	double x=2.5,dx,t=0,dt=.001,p,v,dvol,r=1;
	char c;
	while (x<10)
	{
		t+=dt;
		p=212*(10-x)/144;
		v=0.21*t;
		dvol=(16+10*p-7.5*v)*dt;
		if (dvol<=0)
		break;
		dx=dvol/(3.1417*r*r);
		x+=dx;
		r=(0.4)*x;
	}
	cout<<t<<" "<<"Panama"<<"  "<<dvol<<" "<<x<<"\n";
	cout<<"\n enter c to continue with Peru\n";
	cin>>c;
	x=1,dx,t=0,dt=.001,p,v,dvol,r=0.625;
	while (x<4)
	{
		t+=dt;
		p=50*(4-x)/144;
		v=0.21*t;
		dvol=(3.2+5*p-1.7*v)*dt;
		if (dvol<=0)
		break;
		dx=dvol/(3.1417*r*r);
		x+=dx;
		r=(0.625)*x;
	}
	cout<<t<<" "<<"Peru"<<" "<<dvol<<" "<<x<<"\n";
	return 0;
}
```

The output is:

5.182 Panama 0.00783993 10

enter c to continue with Peru
c
9.409 Peru -1.59207e-007 3.9085

Dunce Hat- Analysis of Simulation Output

Output of the Simulation is:

5.182 Panama 0.00783993 10

enter c to continue with Peru
c
9.409 Peru -1.59207e-007 3.9085

The Earliest Possible Arrival Time for Panama from Time 0 (6:00p)

1 + 16/60 + 4 + 14/60 = 5.5 hours

Time of Empty in Panama = 5.182 hours.

There is not enough time to reach the Panama tank.

The Earliest Possible Arrival Time for Peru from Time 0 (6:00p)

1 + 16/60 + 7 + 5/60 = 8.35 hours.

There will still be fluid in the Peru tank in 9.409 hours.

There is time to stop the leak in the Peru tank.

Note, however, that the leak stops with less than 1/10 of a foot of fluid left in the tank.

The best solution is to "stay home and watch ESPN".

Problem #2: Superhero(ine)

Prince(ss) Dweebo is stationed on the planet Zardoz off Gamma Virgo. To save the universe, he/she must run the ten miles from Ice Station Zebra to Zen Terminal Starfire in 10 minutes. To avoid detection by terminator pods, the trek cannot start until exact sunsset. Initial running speed will be 2 miles per minute. The deceleration (negative acceleration) at any given time will be (0.0711*t) miles per minute per minute (t is time in minutes after sunsset), as the atmosphere of Zardoz turns into Karo syrup from chilling in the dark. Will the universe be saved? How long does the 10 mile run take? If Dweebo plays more video games and waits one minute after sunset before starting, will the universe still be saved? Why or why not?

The code:

```
main()
{
        double t=0,dt=0.001,v=2,dv=0,d=0,dd=0;
        while(d<=10)
        {
                t+=dt;
                v-=dv;
                d+=dd;
                dv=0.0711*t*dt;
                dd=v*dt;
                if(v<=0)
                break;
        }
        cout<<t<<"   "<<v<<"   "<<d<<'\n';
        return 0;
}
```

The output with code as shown:

7.448 0.0282101 10

The output with t starting at one minute:

7.568 -0.000258291 8.24514

Actually, if the trip is delayed just one second, the output will be:

7.50167 -0.000299374 9.96753

Although the trip requires less than 7.5 minutes with 10 minutes allowed, a delay of just one second causes failure to make the trip at all!