All You Ever Wanted to Know about How a Ball Bounces

(But Were Afraid to Ask)

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What happens to a basketball when you drop it? Obviously it bounces, but how high? How long does it take to stop? What mathematical relationships exist? This activity will seek to guide you through these questions and many others.

For this project you will use the calculator-based ranger (CBR) to collect data about the motion of a bouncing ball over time. As a ball bounces its maximum height decreases on each successive bounce. This lab will examine three different aspects of a bouncing ball. First, we will examine the behavior of the peaks of each successive bounce. We will select the maximum height of each bounce and attempt to find a pattern in them. Next, we will examine the position, velocity, and acceleration graphs and attempt to explain the relationships between them. Finally, you will examine a single bounce. The graph of the height of a ball over time for a single bounce should look like a downward opening parabola and therefore be quadratic. You will select one bounce and develop a mathematical model for it. Finally, you will need to take pictures of your group performing the experiment for later use in writing a webpage.

YOU WILL COLLECT THE DATA IN A GROUP,
BUT YOU MUST COMPLETE THIS HANDOUT ON YOUR OWN!

Materials Needed:

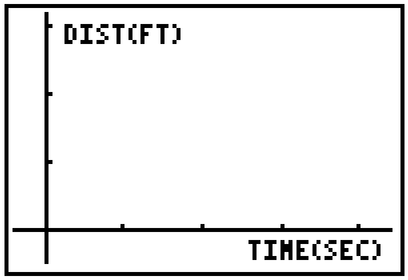
1 CBR Unit	1 Ball
1 TI-83 Calculator	1 Digital Camera

This activity will use all of your standard lists. If you have important data on your calculator you should move it to a named list or store it on your G drive. You will also need to save the data and pictures of this experiment for later use. SAVE THIS INFORMATION TO YOUR ACCOUNT AND DO NOT DELETE IT.

Instructions:

1. Link the CBR to a TI-83.
2. Set the calculator in RECEIVE mode and press the 82/83 button on the CBR. This will send the program RANGER to your 83. It is a very large program; you may need to clear some memory on you calculator.
3. Run RANGER. Select APPLICATIONS from the main menu, then FEET from the next menu.
4. Now choose BALL BOUNCE from the applications menu. Measure the height from which you will drop the ball (in feet)____________.
5. Hold the CBR parallel to the floor and the ball about 1.5 feet below the CBR. Drop the ball and press the TRIGGER button simultaneously. You will hear a series of clicks. Do not move the CBR. Wait until the clicking stops.
6. Reconnect the CBR to the 83. Press ENTER on the 83. Repeat the sample if the data does not look satisfactory.
7. L_1 through L_4 now contain important information you may want to secure this data on your G drive or a named list.

You may need to repeat the activity several times until you get a satisfactory sample. This is an ideal place to take pictures. "Good Data" should look like a series of smaller and smaller parabolas. You need to at least 5 consecutive bounces. Make a rough sketch of your data below.



Relationship between the Rebound Heights

Trace along the graph to find the maximum height of each bounce. Record the maximum height of each bounce in the table below. Round your data to three decimal places.

Bounce Number (*x* value)	**Rebound Height (*y* value)**
0 (**Initial height**)	
1	
2	
3	
4	
5	

You only need data from 5 bounces. If you have more than 5, you may disregard them. Enter the bounce number into L_5 and the heights into L_6. Make a STAT PLOT of this data (use plot that is not connected). Sketch it below and include axes and scale markers. Turn off the plot of L_1 versus L_2.

Questions about the peaks:

1. Compute the ratio between the height of the first bounce and the initial height (divide bounce one height by the initial height). What value did you get? __________

Next divide the height of the second bounce by the height of the first bounce, then the third by the fourth and so on. List the values you got. ____

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2. Theoretically the model for the data recorded above is exponential. In other words the equation is of the from:

$$y = H\,P^x$$

where x is the bounce number and y is the rebound height. In this equation H is the initial height. We know this value:

$$H = \_\_\_\_\_\_\_\_\_\_\_\_$$

Use the equation $y = H\,P^x$ to explain why H is the value of the y-intercept.

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3. Press [Y=] and store the equation $y = H P^x$ in the first available space. Return to the home screen. Next store the value of H into the calculator's memory by typing the number then [STO▶] [APLHA] H [ENTER]. Then you will try to determine the best value for P. Start with $P = 1$, by storing 1 into P in the same manner that you stored the initial height into H. Press GRAPH to display the data and the curve on the same screen. Adjust the value of P until you get a good fit to the data. Record the value of P below:

P = ___________________

What is the real-world meaning of the value determined for P?
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4. Using the equation determined in #2 predict the height on the 6^{th}, 7^{th}, and 8^{th} bounces:

6^{th} bounce height ______________________
7^{th} bounce height ______________________
8^{th} bounce height ______________________

On which bounce will the height of the bounce be less than 10% of the initial height. Remember that the number of bounces must be an integer. Record this value below.

Number of bounces ___________

5. What does the exponential equation tell us about when the ball stops bouncing? Debate the mathematical "truth" against the visual "truth". What have you learned from this portion of the activity?
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Position, Velocity, and Acceleration

Using the STAT PLOT menu create the following plots:

Plot 1: A connected scatterplot between L_1 (Time) and L_2 (Position)
Plot 2: A connected scatterplot between L_1 (Time) and L_3 (Velocity)
Plot 3: A connected scatterplot between L_1 (Time) and L_4 (Acceleration)

Now look at each plot individually and sketch Plots 2 and 3 (Draw axes and scale markers as necessary):

Velocity versus Time graph

Acceleration versus Time graph

Questions about Velocity and Acceleration:

1. Describe the pattern(s) you observe in the Velocity versus time graph.

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2. Describe the pattern(s) you observe in the Acceleration versus Time graph.

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3. Which points on the Velocity graph correspond to the maximum height of each bounce? Defend your answer analytically. (What is the velocity of the ball at it's maximum height?)

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4. What physical event causes the sudden changes each of the graphs?
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5. Some of the values should be negative. Is it possible to have negative velocity and acceleration, or is this equipment simply malfunctioning? Interpret what this may indicate by comparison to the position graphs.
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Analyzing a Single Bounce

Link your calculator to the overhead calculator and transfer the program CHOOSE. This program will allow you to select a portion of the graph. You want to select one parabolic section from your data by running this program. Be sure that you are done with earlier information, as this program will overwrite some of your lists. If you think you may need the earlier data you should save it to another location (not a bad idea). Each member of your group should select a different bounce.

Questions about the bounce:

1. Press TRACE and move the cursor along the graph until you reach the vertex of your parabola (the highest point). Round the numbers to three decimal places and write the numbers below.

x-coordinate	y-coordinate

2. The vertex form of a parabola is

$$y = A(x - h)^2 + k$$

where (h, k) is the vertex, and A is a constant. Store this equation into your equation editor. Return to the home screen and store the values for the vertex in h and k as before. To obtain a good fit for the data, you will have to adjust the value of A. Start with $A = 1$. Press GRAPH to view the data and the curve. Change the value of A until you are satisfied. Write your final value below:

A = ____________

Using your values with the vertex form of your final equation below:

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3. The general form of a quadratic equation is:

$$y = ax^2 + bx + c$$

Expand your equation from the previous question to find values for a, b, and c. Space is provided below for your work:

Write the values for a, b, and c below:

a	
b	
c	

4. Now run a quadratic regression on the data you have collected. Write the values for a, b, and c below:

a	
b	
c	

Are the values of for a, b, and c in the quadratic regression equation above reasonably consistent with the values in your table from question 3? Graph both equations at the same time. Do they look similar? Discuss the pros and cons of each method of finding equations for the data.

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5. In your own words, discuss the effects each constant has on the graph of the equation for:

$$y = a(x - h)^2 + k.$$

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6. What was the most significant thing you learned from this activity?

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