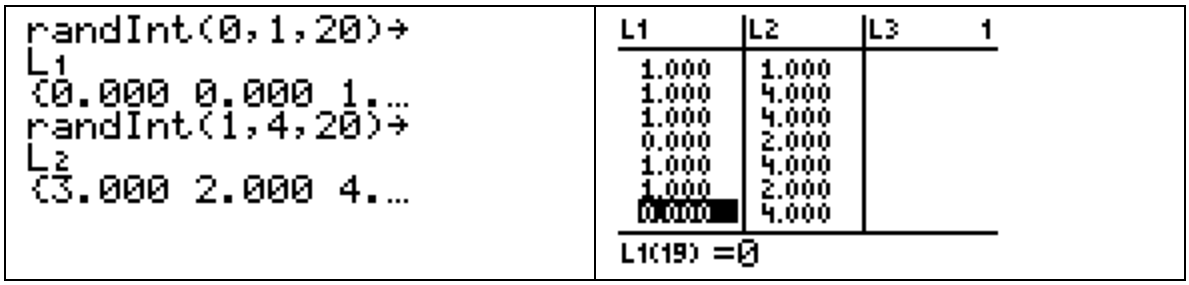
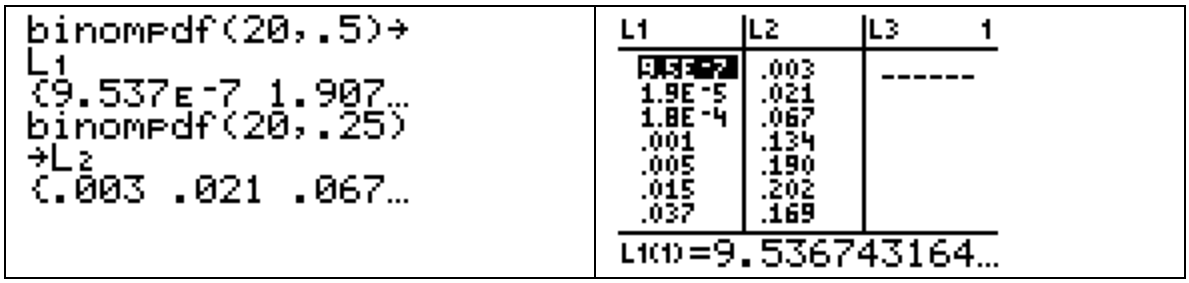
True-False, Multi-choice, what are your chances?
An adventure in binomial distributions.

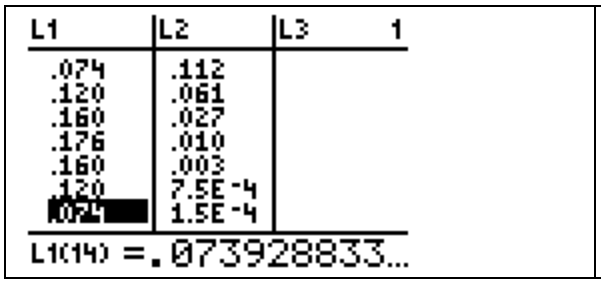
You're a good mathematically inclined professor and you want to make a 20 question true-false test with the answers randomly distributed. So you take your TI83plus and do a **MATH**-*PRB*-5 and finish out the home screen with "0,1,20)? 2^{nd}-L1" and **ENTER**. Now do **STAT**-1 to see the 1's and 0's in list 1. These will be your true's and false's in order. In list 2 you generate the answers in order for a 4 choice multiple choice test.



Now you're a lazy, ah, efficient student about to take such tests. You're going to make better use of your time instead of studying for these tests and you want to make 70% or better with random guessing only. You go through the same exercise the professor performed to generate your answers. So for any one true-false question, you have 1 chance in two. For any one multi-choice question you have one chance in four. Assume you've written down your answers from the lists and we can reuse them. Before we answer the only question you're interested in, mainly, "What are my chances of making 70% or better", let's look the stats over a little. Do **2^{nd}-DISTR**-0 and finish as shown. Then do the same for 1 in 4 probability (.25) as shown. Then do STAT-1 and look over the lists.

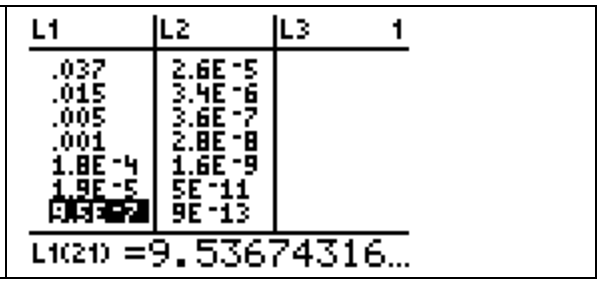


The rest of the lists (L1 and L2) are shown below:



L1	L2	L3 1
.074	.112	
.120	.061	
.160	.027	
.176	.010	
.160	.003	
.120	7.5E-4	
.074	1.5E-4	

L1(14) =.07392883...



L1	L2	L3 1
.037	2.6E-5	
.015	3.4E-6	
.005	3.6E-7	
.001	2.8E-8	
1.8E-4	1.6E-9	
1.9E-5	5E-11	
0	9E-13	

L1(21) =9.53674316...

The first number in the lists is the probability of your getting 0 correct. Fortunately, the probability isn't high. But you have the same probability of getting 20 right. Note that's the 21rst number in the lists. For the true-false, the most likely score is 10 and the probability is the 11th number, 0.176. The most likely score on the multi-choice is 5 and the probability is the 6th number, 0.202.

What range of scores will cover 90% of the probability, i.e., what range of scores can you be 90% certain that your score will fall in? (English majors, reword with less words and so that the last sentence doesn't end in a preposition). In both cases you add up the highest probabilities until you get 0.9 or better, and note the end points, keeping in mind that the first probability listed represents 0 right answers. For true-false, the .5 probability case, the range is 6 to 14 for a 96% chance and 7 to 13 for an 88% chance. For the multi-choice, the range is from 2 to 8 for a 94% chance, 3 to 7 giving an 81% chance.

Now that you've looked that over, let's answer the burning question of what are the chances of making 70% (getting 14 or more right) with almost 0 effort? We could add up the 15th through 21rst numbers in the lists, but let's use something different, the binomial cumulative distribution function. We'll find your chances of scoring 13 or less and subtract from 1 to get your chances of beating the system.

Do **2nd-DISTR**-*A* (alpha or just select it and ENTER)	binomcdf(20,.5,13) .942341 binomcdf(20,.25,13) .999970

Wow. Since these are your chances of making less than 70, maybe you could reset your goal to a 60 or better, or maybe even study just a little.

A person actually made 56 out of 150 on a true false test. What are the chances of a person doing this poorly or worse, even with no knowledge, assuming a random

array of answers? It’s barely over 1/10 of a percent. We should get a grant to study this person’s “system”.