<u>Difference Quotient Activity</u>

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Objectives:

- All students will be able to explain (verbally and/or in writing) the concept of the difference quotient as the definition of the derivative with 100% accuracy.
- All students will be able to evaluate the difference quotient for simple polynomials with 80% accuracy.
- All students will be able to use a TI-92 or TI-89 to define the difference quotient and various functions with 100% accuracy.
- All students will be able to use a TI-92 or TI-89 to evaluate the difference quotient for varying functions (polynomial, trigonometric, exponential, etc.) with 100% accuracy.

Text:

<u>Calculus</u>, by Deborah Hughes-Hallett et al., Chapters 2 & 4

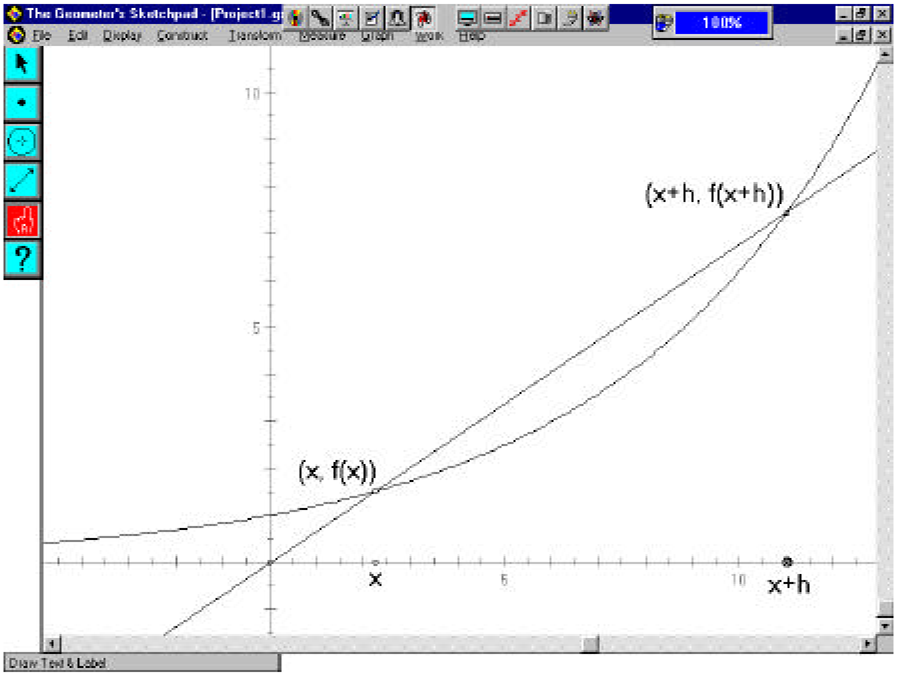
Background:

The concept of the difference quotient is relatively simple for most students to understand. However, the algebraic simplification that usually comes as part of this topic can cause difficulty for even the most gifted math students. This unit is designed to aid students with the complicated algebraic manipulations in order to allow them to focus on concepts more and to discover the rules for the derivatives of simple functions for themselves. For the purposes of this unit it is assumed that the class has already received instruction that speed is the slope of the tangent line. The unit will explore the derivative at a point, the derivative function, and the intuitive development of simple derivative rules.

In the spirit of reform calculus, students should be presented topics according to the rule of four. The four components being: geometrically, numerically, analytically, and verbally. One of the purposes of this lesson is to add a fifth dimension, technologically. The approach from many angles is designed to develop mathematical thinking first then mathematical skills second. Students that have a clear intuitive picture of the central ideas are practicing mathematical thinking. In the traditional classroom, this conceptual understanding is often neglected. It is the purpose of reform calculus and this project to develop this level of thinking. The added use of technology serves to free students from performing challenging paper and pencil manipulations to greater enhance their level of mathematical thinking. After students develop understanding of a concept the focus naturally changes to mathematical skills, which will become strengthened by repeated practice.

The Lesson:

We begin by discussing the notion of the derivative as the slope of the tangent line at a point. Since we cannot find the slope with just one point we use a technique similar to measuring speed. We take short intervals to approximate the value. If we have a function we can make the interval very small and therefore more accurate. Illustrate this dynamically with the Geometer's Sketchpad illustration below.



Next we need to find the slope of the secant line in the given picture. The results follow.

$$Slope = \frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

This, of course, is the difference quotient. The next step is to shrink the value for h. This leads naturally to the notion of a limit. (Reform calculus does not teach limits as a separate topic, it is integrated throughout the entire text.) The addition of a limit is the final step in the definition of the derivative at a point (or more generally of an entire function).

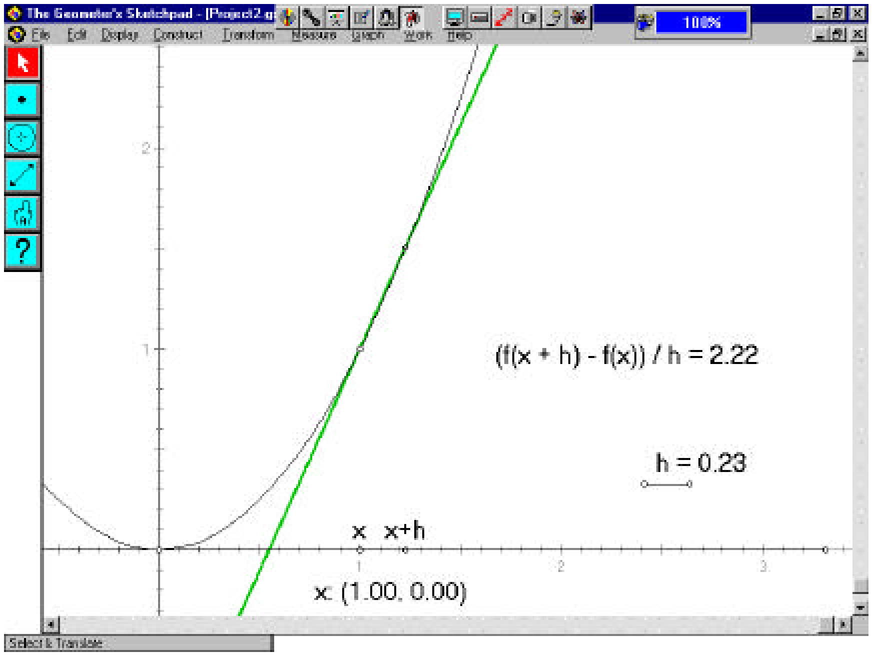
$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Next we use this definition to find the derivative of $f(x) = x^2$ at $x = 1$. I divide students in small groups and have each group investigate this problem in one of four ways: using dynamic geometry, analytically using the difference quotient, using the TI-83 tables, or using graph paper and spaghetti. Each group should share their results when finished. Examples of each follow:

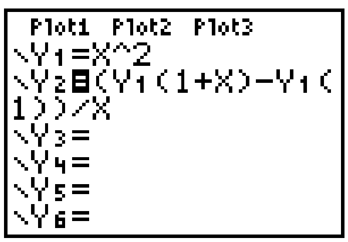
Analytic Computation

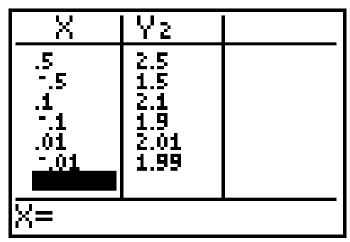
$$f'(1) = \lim_{h \to 0} \frac{(1+h)^2 - 1^2}{h} = \lim_{h \to 0} \frac{1+2h+h^2-1}{h} = \lim_{h \to 0} \frac{2h+h^2}{h} = \lim_{h \to 0} 2+h = 2$$

Dynamic Geometry



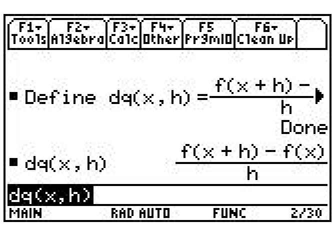
TI-83 Tables

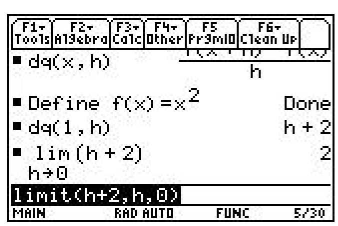


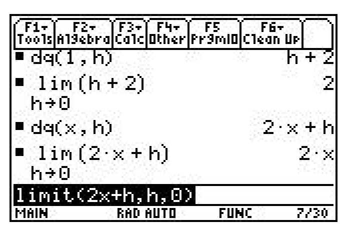


In all cases the answer should work out to approximately two. We then discuss the pros and cons of each method. The analytic solution is the only exact one, but it is perhaps the most difficult to work.

Next we add the TI-92 or TI-89 to automate the simplification of the difference quotient. First define the difference quotient using the Define command from the Other menu (F4) from the home screen (see **dq** function).







The new difference quotient function can be used to evaluate any function at any point or to find the derivative function. The limit can be incorporated into the definition of the function (see the attached **der** function for the TI-89), but I prefer to have students evaluate the limit initially. Also, it is relatively easy to define the symmetric difference quotient, a topic usually avoided because it complicates expressions. This topic is usually assigned as homework.

Name ______________________________

Difference Quotient Homework

1. Find the derivative of $f(x) = 3x^2 - 8x$ at $x = 3$
   a. Using dynamic geometry
   b. Using the defined difference quotient on your TI-92 or TI-89
   c. Analytically using paper and pencil
   d. Discuss the pros and cons of each method.

2. Another way to define the derivative is using the symmetric difference quotient. This method does not use the function value at the point in question. Instead it uses values above and below the given point as follows:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

Define this function in your calculator as **sdq** and use it and the original definition of the difference quotient to evaluate the following. Compare the results.
   a. Find the derivative of $f(x) = x^2$ at $x = 3$.
   b. Find the derivative of $f(x) = \sin x$ at $x = p/4$
   c. Find the derivative of $f(x) = 3x^5$

3. Define a difference quotient that incorporates a limit. Name this function the **der**. Use it to evaluate the following.
   a. The derivative of $f(x) = e^x$ at $x = 0$ and $x = 1$
   b. The derivative of $f(x) = 2x^5 - 6x^4 + 2x^3 - 8x^2 - 3x + 5$ at $x = 2$
   c. The derivative of $f(x) = 2x^5 - 6x^4 + 2x^3 - 8x^2 - 3x + 5$

4. Use the derivative function defined in problem 3 to find the following.
   a. $f(x) = x^2$
   b. $f(x) = x^3$
   c. $f(x) = x^4$
   d. $f(x) = x^5$
   e. $f(x) = x^6$
   f. $f(x) = x^{20}$
   g. $f(x) = x^{50}$

5. Using your results of problem 4 make a conjecture about the derivative of $f(x) = x^n$.

6. Find the derivative function of the following.
   a. $f(x) = e^x$
   b. $f(x) = 2^x$
   c. $f(x) = \sin x$
   d. $f(x) = \cos x$
   e. $f(x) = \ln x$