

MATHEMATICS TEACHING UNIT

LINEAR FUNCTIONS

- 1. What does slope really mean?**
- 2. Gathering linear data**
- 3. Let's Graph!**

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UNIT: LINEAR FUNCTIONS

LESSON: What does slope really mean?

OBJECTIVE: Students will make the connection between a real-life linear model and the concept of slope as a constant rate of change.

Materials Needed:

Styrofoam cups – four to eight for each student pair
Rulers – one for each student pair
Paper and pencils

Teaching Strategies:

1. Divide students into pairs.
2. Have Students clear their desks except for paper and pencil.
3. Distribute to each student pair several Styrofoam cups and one ruler.
4. Have student pairs measure the vertical height of one cup and record.
5. Have student pairs stack another cup into their first cup, measure the vertical height of two cups and record.
6. Continue process until each student pair has data for several cups heights.
7. Discuss each pair's consistency in the change from measurements of one cup to two cups and from two cups to three, etc. Note that the change is the measurement of the band around the top of the cups. Relate that this consistent rate of change can also be viewed as slope.
8. Introduce or reintroduce the point-slope form of a line.
9. Have each pair choose two of their data points and find the equation of their line that represents their "cup" data.
10. Compare different student pairs' equations. Discuss why each equation might be very close to each other and what could account for differences.

UNIT: LINEAR FUNCTIONS

LESSON: Gathering Linear Data

Objectives:

- ? **Students will be able to name the other students in their class and know where they are from**
- ? **Students will be able to enter and analyze linear data in their graphing calculator using the statistical capabilities of the graphing calculator**
- ? **Students will be able to analytically find the linear equation of gathered data**

Materials Needed:

Overhead Graphing Calculator with projection panel for instructor
Graphing Calculators for each student
Two Stop Watches
Paper and Pencils

Lesson Narrative:

I use one of my favorite teaching activities on the first day of class for junior precalculus students. High school juniors and seniors attending Arkansas School for Mathematics and Sciences (ASMS) come from all across Arkansas. Very few junior students know any other students attending ASMS. They come from large schools, small schools, and inter-city schools. They come from schools with strong mathematics programs and some with struggling mathematics programs. Most students have a high mathematical ability, however they may not have had the opportunity to develop it. These students attend their first mathematics class at ASMS generally feeling insecure, scared, and even intimidated by the newness of being away from home for the first time.

With this in mind the “Gathering Linear Data” lesson has several objectives. One objective is to give students the opportunity of introducing themselves and where they are from *I feel that it is important that they know the faculty at ASMS respects the students’ backgrounds and home schools. Another objective is to introduce them briefly to some of the statistical capabilities of the graphing calculator. Some students have used graphing calculators. At least 50% have had very limited to no exposure to graphing calculators. Finally, this activity incorporates mathematics topics that most precalculus students are familiar with. This supports the idea that the students do have the mathematical skill and understanding to be successful ASMS mathematics students.

Two student are given stop watches and are given the role as “timers”. Another student is designated as the time “averager”. This student’s responsibility is to add the times from the two “timers” and average them. Another student is given the responsibility of recording, on a T-chart on the overhead, data of two variables: number of students and times. The first student says “start” and then tells his/her name and where he/she is from and then says “stop”. The timers give their times to the averager who averages the times and the recorder records the data. This process may involve a few false starts as students become familiar with the stop watches and the process. It is also an opportunity for students to laugh and enjoy mathematics. I then add additional students making a group of 3 or 4 students. The first student again says “start”, tells his/her name and where he/she is from, as do the other students in the other group.

The last student in the group says “stop”. The timers time, the averager averages, and the recorder records. I then select a larger group of students which includes the first group. This process is repeated until all students are participating and 5 to 8 sets of data have been collected**

We then discuss what the graph of this data might look like and why. I interject the possibility of the data being exponential or quadratic. Some students may be unsure of the meanings of these terms but it gives them the idea that perhaps they will soon learn. It is usually agreed that if the variables were the same that the data would be linear. We discuss slope, how it is found given data points, and its relevance to our data. This is perhaps the first time some students have made a connection between real data and what the slope of a line means. I then ask how we might find an equation that closely represents our data. Usually at least one student remembers the “Point-Slope” form of a line. We then discuss how this might give us an equation. At this time most students are remembering some mathematics from their previous mathematics classes from their home schools. This in itself adds to their comfort level. The students then analytically find an equation. We discuss why all the students’ equations are not identical and why this may or may not be relevant.

Students have either brought their own TI-83 or TI-83 Plus graphing calculator or they have checked one out through the mathematics department. Using the statistical capabilities of the TI-83, I share with students how to put our data into two lists and using Stat Plot, graph the data. We then graph the equation of the line that they previously found analytically and discuss its accuracy. We then do a linear regression on the data and compute an equation using the TI-83. We graph this equation and compare it to the previous graph. We discuss the differences and similarities of the graphs and their relevance.

During this activity, questions are being asked continually. Students are expected to be engaged and their ideas are openly discussed and analyzed. It is their first insight into what they can expect in my classroom. They are expected to be an active learner. They are expected to voice their opinions and be able to defend them. All students are encouraged to respond and share. Usually before this activity is over I have faded into the background and students are sharing, debating, and investigating the problem situation. For evaluation, I ask students to write a Reflection Paper for the “Gathering Linear Data” lesson which demonstrates mathematical skill as well as communication. This is a first glimpse of my writing and communicating expectations.

This “Gathering Linear Data” lesson has been successful as a first day mathematical introduction for Precalculus and Algebra II students. They realize that they do have prior mathematical skill, that they are not alone in feeling apprehension in a new situation, that they are not alone in their limited calculator knowledge, and that they are going to enjoy being so actively engaged in their mathematics class. I have found that this activity is an exciting way to begin a new school year.

* I have also used this activity in teaching university instructors how to incorporate the calculator into their teaching. Instead of collecting data on where the instructors were from (they were all from the same town), we collected data on where they were born.

** A challenging variation of this activity is for each student to not only give their name where they are from but also to give the names and home towns of all other students called on prior to them. For example, the last student in the class to be called on would be responsible for naming all the students in the class and where they are from. Obviously the graph of this data will not be linear.

UNIT: LINEAR FUNCTIONS

LESSON: Let's Graph!

OBJECTIVES:

- **Students will begin to feel comfortable using the graphing calculator.**
- **Students will be able to identify graphs with large and small slopes.**
- **Students will be able to identify vertical change in graphs.**
- **Students will be able to graph linear functions with a graphing calculator.**

Materials Needed:

- Overhead Graphing Calculator with projection panel (This lesson is written with the TI-83 or TI-83 Plus Graphing Calculator in mind).
- Graphing Calculators for each student.
- Graph Paper
- Straight Edges
- Pencils

Teaching Strategies:

- I. Beginning Graphing
 1. Have students clear their desks except for a graphing calculator, paper and pencil.
 2. Introduce students to the following keys and discuss their functions:
 - a. $Y =$, At this time make sure all calculator entries are blank
 - b. Window
 - c. Zoom – Standard and Decimal will be the most common used
 - d. Mode – Make sure calculators are in function mode and floating decimal – Actually best if all choices on the left are selected (dark)
 - e. Stat Plot – Make sure all are turned off
 - f. Quit – Studnets can always return to the Home Screen by using this key
 3. At Home Screen, demonstrate and have students do simple arithmetic calculations, emphasizing order of operations
 4. Have students graph the following functions, discussing the reasons for the graphs' changes. Following this discussion, change the positive x-coefficients to negative and discuss the graphs' changes. Be sure to use the appropriate terminology of slope and y-intercept. Revise the windows and discuss these changes.

$$y = x \qquad y = 2x$$

$$y = \frac{1}{2}x \qquad y = x + 2$$

$$y = x - 3 \qquad y = 2x + 1$$

$$y = 5x - 4 \qquad y = 2x - 6$$

II. Using a Table

1. Have students put the function $y = 3x - 4$ into $y =$. Investigate the graph and discuss its slope and y-intercept.
2. Introduce the TRACE key and discuss how this key may not always give the values that are needed.
3. Revise the window, graph and trace again. Note changes in traced values.
4. Under CALC key, use the value and zero choices. Have students analytically find a value and zero and compare their results to the calculator-found values. Discuss the reasons for minor differences, emphasizing that the calculator is just a tool and may not always give exact solutions, i.e., instead of giving zero as the correct solution to a problem, the calculator may give .0000000000012.
5. Have students go to the TBLSET key. Discuss what TblStart value the class will use as well as Δ Tbl value.
6. Have students go to the TABLE key. Discuss what the values mean and what a great feature this is of the calculator and how it can be such a time saver.
7. Have students graph the function on graph paper, using their calculator graph as a guide. Emphasize that all graphs must have axis labeled as well as the scales.

III. Making Conjectures

1. Discuss the uses of the graphing calculator as students begin their mathematical journey. Emphasize that the calculator is a teaching tool and will be used for verification and NOT as proof. Students are and will always be required to be able to analytically find mathematical solutions.
2. Extend the activity to introduce quadratic functions and their characteristics.